

Fig. 3 Simulation results for the step and time-varying commands.

### Conclusions

A model-following pitch autopilot using adaptive quasi-sliding mode control for a sampled-data system with model uncertainties and disturbances has been presented. The unknown parameters, which need not satisfy the matching conditions and the unknown disturbances and whose upper bound need not be known, are compensated for by applying an on-line adaptive algorithm. The proposed controller shows the asymptotic tracking of the pitch acceleration of the reference model and is able to provide robust performance of the system to the model uncertainties and disturbances.

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## Hybrid Fuzzy Sliding-Mode Control of an Aeroelastic System

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### Introduction

GROUND and flight tests of several advanced high-performance aircraft have exhibited a variety of nonlinear aeroelastic responses, including limit-cycle oscillation (LCO) and even instability.<sup>1,2</sup> Recently, a series of papers have considered the control of the aeroelastic system.<sup>2–5</sup> Most of the preceding papers consider the single-input, single-output aeroelastic control problem, that is, flap deflection has been used to control the pitch angle, and the plunge displacement has been shown to be asymptotically stable without control<sup>2,3</sup>; or the plunge displacement has been controlled by the flap deflection, and the pitch angle has been shown to be asymptotically stable without control.<sup>4,5</sup> Moreover, most of these design methods require a system model and complex design procedures. However, the modeling of an aeroelastic system is a work of approximation, and the precise model of an aeroelastic system can be difficult to formulate.

Fuzzy control using linguistic information can model the qualitative aspects of human knowledge. Fuzzy control also possesses several advantages such as robustness, model-free and rule-based algorithm.<sup>6</sup> However, the huge amount of fuzzy rules makes the analysis complex. Some researchers have proposed fuzzy sliding-mode control design methods to reduce the fuzzy control rules.<sup>7,8</sup> This Note proposes a hybrid fuzzy sliding-mode control (HFSMC) scheme for a nonlinear aeroelastic system. The aeroelastic system can be represented as two second-order subsystems that represent the plunge and pitch motions, respectively. By using the sliding-mode control, each subsystem can be controlled in terms of a corresponding sliding surface. A hybrid sliding surface, which includes two subsystems' information, is defined to generate a control effort to make the state trajectories of both subsystems move toward their sliding surface and then simultaneously approach zeros. A comparison between an adaptive control and the proposed HFSMC for an aeroelastic system is presented. Simulation results indicate that the proposed HFSMC can achieve superior control responses for the simultaneous control of plunge and pitch motions.

### Nonlinear Aeroelastic Control System

A prototypical aeroelastic wing section is shown in Fig. 1. Define the state variables and the control input as

$$\mathbf{x} = [x_1 \ x_2 \ x_3 \ x_4]^T = [h \ \dot{h} \ \alpha \ \dot{\alpha}]^T \quad (1)$$

$$u = \beta \quad (2)$$

where  $h$  is the plunge displacement,  $\alpha$  is the pitch angle, and  $\beta$  is the flap deflection. The equations of the motion can be written as

$$\begin{aligned} \dot{x}_1 &= x_2, & \dot{x}_2 &= f_1(\mathbf{x}) + g_1 u \\ \dot{x}_3 &= x_4, & \dot{x}_4 &= f_2(\mathbf{x}) + g_2 u \end{aligned} \quad (3)$$

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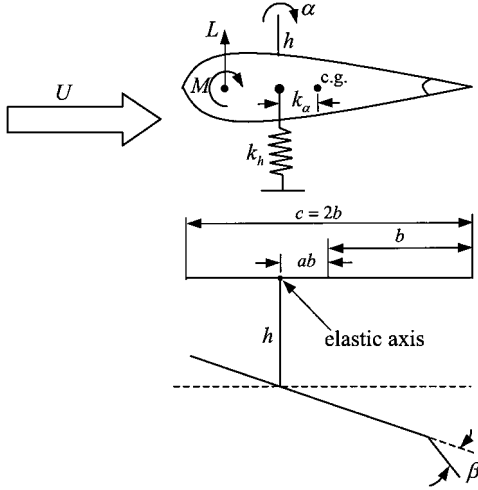


Fig. 1 Aeroelastic model.

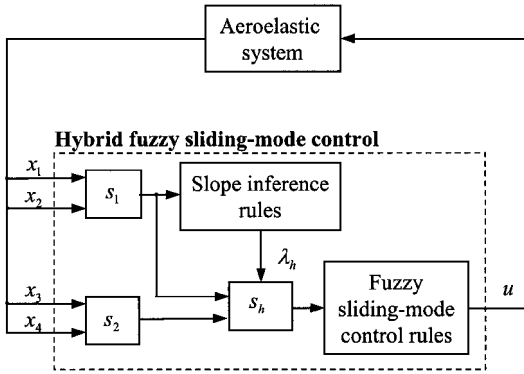


Fig. 2 Block diagram of hybrid fuzzy sliding-mode control aeroelastic system.

where

$$\begin{aligned} f_1(\mathbf{x}) &= -k_1 x_1 - [k_2 U^2 + p(x_3)] x_2 - c_1 x_3 - c_2 x_4 \\ f_2(\mathbf{x}) &= -k_3 x_1 - [k_4 U^2 + q(x_3)] x_2 - c_3 x_3 - c_4 x_4 \\ g_1 &= -(I_{\alpha} \rho b c_{l_{\beta}} + m_w x_{\alpha} \rho b^3 c_{m_{\beta}}) U^2 / d \\ g_2 &= (m_w x_{\alpha} \rho b^2 c_{l_{\beta}} + m_T \rho b^2 c_{m_{\beta}}) U^2 / d \end{aligned} \quad (4)$$

and the expressions for the parameters are given by Strganac et al.<sup>2</sup>

### Hybrid Fuzzy Sliding-Mode Control

A HFSMC is proposed to control the whole system states to approach to zeros with satisfactory transient responses. Define a hybrid sliding surface as

$$s_h = s_2 + \lambda_h s_1 \quad (5)$$

where  $s_1 = x_2 + \lambda_1 x_1$ ;  $s_2 = x_4 + \lambda_2 x_3$ ; and  $\lambda_1, \lambda_2$ , and  $\lambda_h$  are positive constants. From the sliding-mode control viewpoint<sup>7</sup> the slope of the hybrid sliding surface  $\lambda_h$  will govern the transient responses of the states. The control objective of the HFSMC is to let the hybrid sliding surface  $s_h$  approach zero. In this case the sliding surface variables  $s_1$  and  $s_2$  will simultaneously converge to zeros, and then the two subsystems  $(x_1, x_2)$  and  $(x_3, x_4)$  will also converge to zeros simultaneously. A principal diagram for HFSMC is given in Fig. 2, which includes two fuzzy inference systems, the slope inference rules, and the fuzzy sliding-mode control rules. The slope of the hybrid sliding surface  $\lambda_h$  will play an important role in governing the interactive transient responses between the states of these two subsystems. The slope inference rules are expressed as

$$\text{Rule } i: \text{ If } s_1 \text{ is } F_1^i, \quad \text{then } \lambda_h \text{ is } \Theta_i \quad (6)$$

**Table 1 Fuzzy rules for the slope of the hybrid sliding surface**

$S_1$	$\Theta_i$
NB	0.1
NM	2.5
NS	10.0
ZO	20.0
PS	10.0
PM	2.5
PB	0.1

**Table 2 Fuzzy rules for the control action**

$S_h$	$\Omega_j$
NB	-0.20
NM	-0.15
NS	-0.07
ZO	0
PS	0.07
PM	0.15
PB	0.20

where  $F_1^i$ ,  $i = 1, 2, \dots, n$  are the labels of the fuzzy sets characterized by the fuzzy membership functions  $\mu_{F_1^i}(\cdot)$  and  $\Theta_i$ ,  $i = 1, 2, \dots, n$  are the singleton hybrid sliding surface slope given in Table 1. This slope inference system is constructed by the idea of decreasing trajectory convergence time. The fuzzy sliding-mode control rules are designed as

$$\text{Rule } j: \text{ If } s_h \text{ is } F_2^j, \quad \text{then } u \text{ is } \Omega_j \quad (7)$$

where  $F_2^j$ ,  $j = 1, 2, \dots, m$  are the labels of the fuzzy sets characterized by the fuzzy membership functions  $\mu_{F_2^j}(\cdot)$  and  $\Omega_j$ ,  $j = 1, 2, \dots, m$  are the singleton control actions given in Table 2. The singleton fuzzy sliding-mode control rules are constructed using the idea that the state can quickly reach the hybrid sliding surface without large overshoot. The fuzzy labels used in this study are negative big (NB), negative medium (NM), negative small (NS), zero (ZO), positive small (PS), positive medium (PM), and positive big (PB). The defuzzification of the output is accomplished by the method of center of gravity:

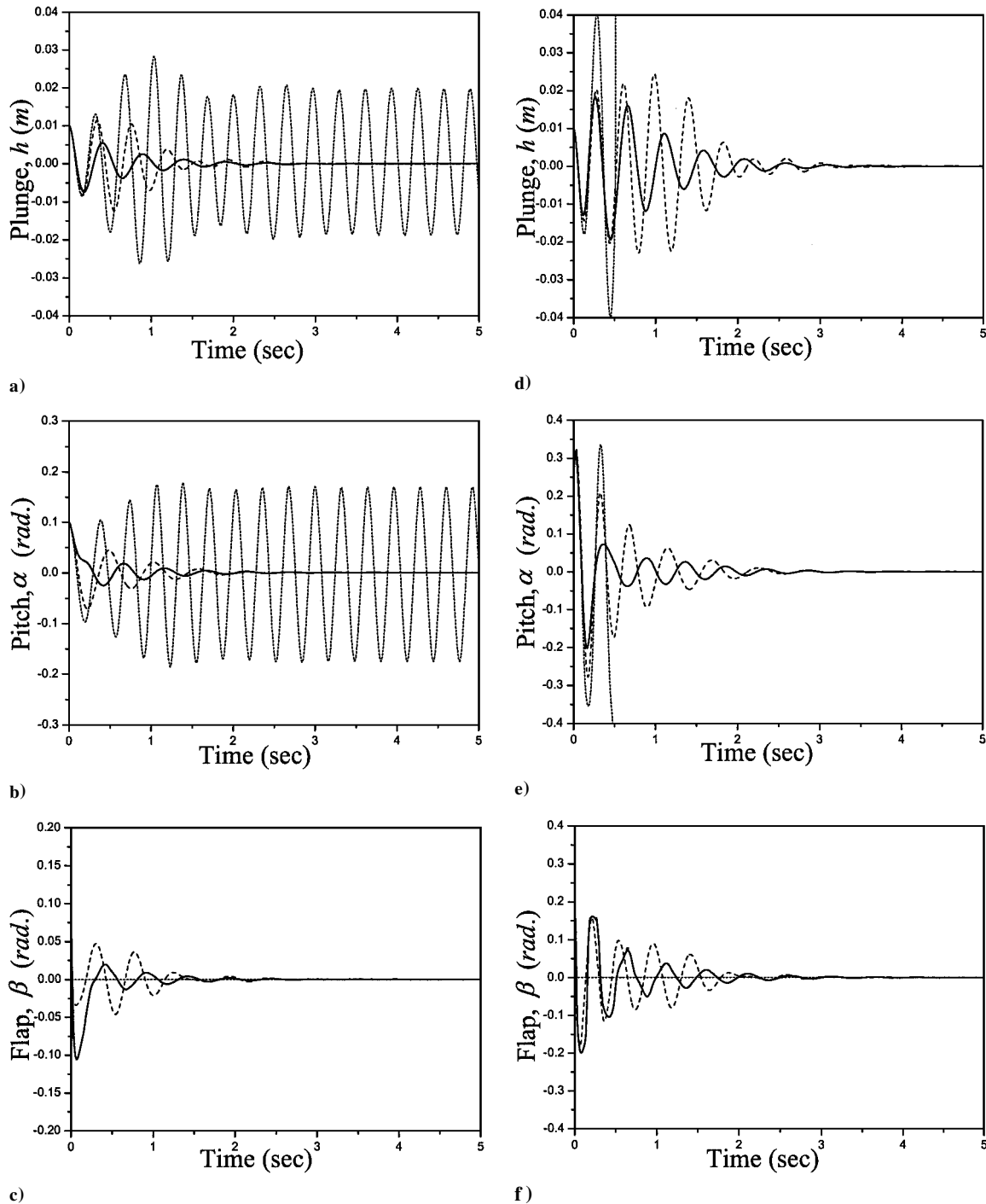
$$\lambda_h = \frac{\sum_{i=1}^n \xi_i \times \Theta_i}{\sum_{i=1}^n \xi_i} \quad (8)$$

$$u = \frac{\sum_{j=1}^m v_j \times \Omega_j}{\sum_{j=1}^m v_j} \quad (9)$$

where  $\xi_i = \mu_{F_1^i}(s_1)$  and  $v_j = \mu_{F_2^j}(s_h)$  are the firing weights of the  $i$ th and  $j$ th rules of Eqs. (6) and (7), respectively.

### Simulation Results

In this section numerical results for the plunge displacement and pitch angle control are presented. It should be emphasized that the derivation of the proposed HFSMC does not need to use the aeroelastic system model in Eq. (4). The design of HFSMC is based on the assumption that all of the states ( $h$ ,  $\dot{h}$ ,  $\alpha$ , and  $\dot{\alpha}$ ) are available. To investigate the effectiveness of the proposed HFSMC, two open-loop cases—a LCO case [ $h(0) = 0.01$ ,  $\dot{h}(0) = 0$ ,  $\alpha(0) = 0.1$ , and  $\dot{\alpha}(0) = 0$ ] and an unstable case [ $h(0) = 0.01$ ,  $\dot{h}(0) = 0$ ,  $\alpha(0) = 0.25$ , and  $\dot{\alpha}(0) = 0$ ]—were considered. For comparison, an adaptive control technique (ADC) presented in Ref. 5 is also applied for the aeroelastic control system. The simulation results using HFSMC and ADC are shown in Fig. 3. The plunge displacement responses are shown in Figs. 3a and 3d, the pitch angle responses are shown in Figs. 3b and 3e and the associated control efforts are shown



**Fig. 3** Time responses of the aeroelastic system for a limit-cycle oscillation case and an unstable case: (—, hybrid fuzzy sliding-mode control; ---, adaptive control; and ···, uncontrol).

in Figs. 3c and 3f for a LCO case and an unstable case, respectively. Simulation results show that the control performance using the HFSMC is better than using the ADC. Thus, the HFSMC is more suitable for the aeroelastic control by simultaneously control the plunge displacement and pitch angle.

### Conclusions

In this Note, a hybrid fuzzy sliding-mode control (HFSMC) of an aeroelastic system is proposed. The idea behind the control law is as follows. First, the aeroelastic system is treated as two subsystems such that each subsystem has a separate control target expressed in terms of a sliding surface. Then, a hybrid sliding surface, which includes two subsystems' information, is defined to generate a control effort to make both subsystems move toward their sliding surface. The proposed HFSMC comprises two fuzzy inference systems. The slope of the hybrid sliding surface is tuned by a slope

inference system, then the behavior of the states is regulated by a fuzzy sliding-mode control system. By comparing with the adaptive control technique, the proposed HFSMC, which simultaneously controls the plunge displacement and pitch angle of an aeroelastic system, can achieve better control performance.

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